Magnetohydrodynamic Species Separation in a Gaseous Nuclear Rocket

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This analytical study deals with the possibility of steady flow separation of U^{235} from hydrogen by magnetohydrodynamically spinning the fluid in a cylindrical vortex A gaseous nuclear rocket configuration is described and its model analyzed The hydromagnetic equations are solved for the velocity distribution, pressure field, concentration of uranium and hydrogen, electrical power required, and dissipation rates as a function of Hartmann number and radial Reynold's number Some properties of a hydrogen plasma at a temperature of 1 ev are presented and potential rocket performance given

Nomenclature

B	=	magnetic induction
B_0	=	applied axial magnetic field
b	=	critical buckling
D	=	ordinary coefficient of diffusion
E	=	electric field intensity
G_0	=	radial mass flow rate (mass/area-sec)
h_0	=	half length of cylinder
I_r	=	total radial electric current
J	=	electric current density
k	=	Boltzmann constant
L	=	diffusion length of thermal neutrons
m	=	mass of particle (if subscripted) or a dimensionless constant [defined in Eq (35)]
M	=	Hartmann number $[B_0r_0(\sigma/\eta)^{1/2}]$ or critical mass (if subscripted)
n	=	number density
p	=	pressure
P	=	total electric power
P_m	=	total electric power input per unit mass
Q_j	=	joule heating
Q	=	viscous dissipation rate
r,θ,z	=	radial, azimuthal, and axial positions
r_1	=	radius of inner electrode
r_0	=	radius of outer electrode
R	=	cavity radius
R_N	=	radial Reynolds number (G_0r_0/η)
t	=	time
T	=	temperature
u, V_θ, w	=	velocity components in r, θ, and z directions
V	=	velocity
$\bar{V}$	=	volume
z	=	axial position
ϕ_0	=	electrical potential difference applied across the electrodes
ρ	=	density
η	=	absolute coefficient of viscosity
σ	=	electrical conductivity or thermal neutron absorption cross section (if subscripted)
μ	=	magnetic permeability
τ	=	Fermi age of neutrons in gaseous hydrogen moderator
λ	=	mean free path
ζ	=	r/r_0
ζ_1	≡	r_1/r_0
ξ	≡	z/r_0
α	≡	m_H/m_U
α_p	≡	m_U/m_P where $m_P \equiv \sum_i n_0\, m_i / \sum_i n_{0i}$
Π	≡	$P_e/(4\pi h_0 r_0 G_0^3/\rho^2)$
δ	≡	$r_0G_0m_2/\rho D m_P$
ϵ_0	≡	$(m_2G_0^2/2\rho^2)/kT$
β	≡	$(G_0/\rho)/(\phi_0/B_0r_0)$

Subscripts

0	=	quantity at the outer wall or constant quantity
1	=	quantity at the inner wall or quantity referred to species 1 (hydrogen)
2	=	quantity referred to species 2 (uranium 235)
H	=	quantity referred to hydrogen
U	=	quantity referred to uranium 235
r,θ,z	=	component quantity along coordinate axis

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I Introduction

IN the search for large thrust rockets with high specific impulse, the potential advantages of nuclear rockets are well documented [1–4] The current nuclear rocket program in the U S is developing a solid-core heat-transfer type, hydrogen fueled, nuclear rocket This rocket will have a specific impulse of two to three times that of chemical rockets Beyond this, the high-temperature materials problem looks too formidable for large increases in specific impulse using the solid-core, heat-transfer type nuclear rocket

The concept of a gaseous core nuclear rocket has been dis cussed previously by Shepherd,[5] Safonov,[6] Bussard,[1] Kerrebrock and Meghreblian,[7] Rom and Ragsdale,[8] Meghreblian,[3 4] and others [2] Gaseous reactors may permit the attainment of very high temperatures (at least several election volts) and hence very high specific impulse together with large thrust There are, however, several very difficult technical problems These are primarily as follows:

1) The design of a critical gaseous reactor of sufficiently small volume and mass to be considered for space flight

2) The separation of the uranium fuel from the hydrogen propellant in the gas phase so that the valuable fuel is not steadily lost from the reactor at an excessive rate

3) The efficient energy transfer from the fission products to the hydrogen propellant within the gaseous core

4) The energy transfer from the gas phase of the reactor to the containing rocket walls must be kept within technically tolerable limits for the materials and time duration needed

The design of a purely gaseous nonreflecting critical reactor using U^{235} and hydrogen gas as moderator requires a gas pressure of thousands of atmospheres[1 5 8] for reasonable volumes and hence would involve unobtainably high temperature structures The use of a moderating reflecting wall and a cavity type reactor[6 8] appears to permit a reasonable size flight type reactor

The separation of most of the uranium fuel (U^{235}) from the hydrogen propellant is needed for any practical rocket

Table 1 Distribution of fission energy

Fission energy	Mev
Kinetic energy of fission fragments	168
Gamma ray energy (instantaneous)	5
Kinetic energy of fission neutrons	5
Beta and gamma rays	13
Neutrons	10
	201 total

system A vortex, centrifuge type system has been suggested by Kerrebrock and Meghreblian[7] using peripheral gas jets to create and maintain the vortex Reviewers of this paper kindly brought to our attention the relevant work on magnetohydrodynamic driven vortices by Romero,[10] Keyes,[22] and Lewellen [23] Some fluid mechanical experiments to check the feasibility of vortex containment have been carried out and analyzed by Rosenzweig et al ,[11] Ragsdale,[12] Kendall,[13] and Romero [14] The influence of turbulence and effects of secondary flows are very important in vortex systems [9]

The energy transfer from the fission products to the hydrogen gas will take place primarily by collisions between fission fragments and hydrogen atoms The distribution of fission energy is given approximately in Table 1 The major contribution to dissociating and heating the hydrogen will come from the fission fragments, which fortunately accounts for the majority of the fission energy

The energy transfer from the hot gas to the reactor walls is a complex subject The radiation losses will depend upon the optical depth of the gas filled cavity Cold hydrogen flowing through the wall in a radial direction will help cool the structure, but the exact heat-transfer balance between cool propellant and convective and radiant heat transfer from the hot reactor gas core requires a more definitive structure configuration and reactor design than is yet available

II Magnetohydrodynamic Configuration

In what follows we have analyzed the possibility of creating and maintaining the separation vortex by magnetohydrodynamic means This hydromagnetic vortex in a cylindrical geometry employs an externally applied longitudinal magnetic field and a radial electric field A review of high-temperature rotating plasma experiments, primarily concerned with the thermonuclear program, has been given by Wilcox [15] There is considerable experimental evi dence that such techniques can generate very high speed rotation (azimuthal velocities to 10^8 cm/sec), but the details of the flow field are not well understood Plasma experimental devices called homopolar, xion, ion magnetron, and others employ plasma rotation

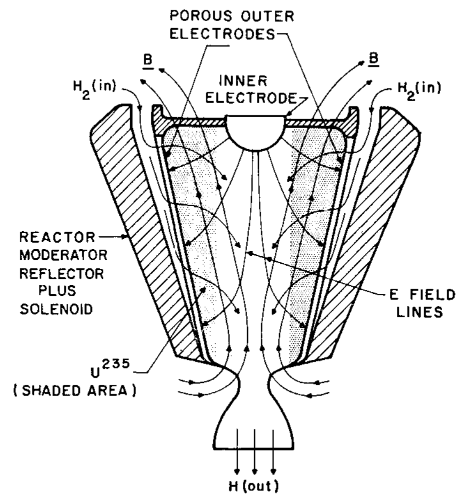

Fig 1 Gaseous nuclear rocket configuration

Consider a configuration for the gaseous nuclear rocket as shown in Fig 1 The rocket "combustion" chamber consists of a nuclear cavity with cylindrical sloping sides At the upper end is an electrode that is separated from the conducting side walls (which are porous) by insulators The electric field lines, designated by E, are seen to have a strong radial component An externally applied magnetic field B is produced by a solenoid and the B lines are nearly perpendicular to the electric field The reactor reflector made of either Be or C, surrounds the reactor For a 6-ft-diam cavity, an amount of about 14 kg of U^{235} is required for criticality [6] Cold hydrogen gas is forced radially through the outer wall Upon passing through the reactor it is dissociated and heated by energy transfer from the fissioning U^{235} The average gas temperature in the core is high enough to pro duce some electrons which, in turn, make the gas a dilute plasma Current flowing from the inner to the outer electrode interacts with the B field to produce an azimuthal Lorentz force causing the gas to spin The centrifugal force field keeps the heavy U^{235} in the outer part of the reactor The nozzle throat at the bottom center causes a flow of material, primarily hot hydrogen, out of the reactor The nozzle accelerates the relatively high-pressure hydrogen to high velocity and, hence, high specific impulse

The nozzle throat might have a small auxiliary magnetic field to help reduce the throat heat transfer, or it may be cooled by cold H_2, or it may be made of ablative material The outer wall will be cooled by the inflow of cold hydrogen

The configuration shown in Fig 1 is too complex to start an analysis of the behavior of such a system Instead, consider a simple coaxial long cylinder with a uniformly applied longitudinal magnetic field and a radially applied electric field as shown in Fig 2 The plasma is contained within the coaxial cylinders and is made to rotate by the Lorentz force resulting from the radial current and the applied longitudinal field The Lorentz force is counterbalanced by the viscous effects of the plasma If there were no viscosity, a steady state is still possible wherein the plasma will accelerate until the induced electric field ($\mathbf{V} \times \mathbf{B}$) cancels the applied field $\mathbf{E}$ and the current (and hence Lorentz force) goes to zero in a closed cylindrical system Imagine that the coaxial cylinders are porous so that a cold gas (hydrogen) can be radially fed into the plasma (reactor) space and withdrawn from the inner cylinder as hot plasma If the gas within the coaxial cylinders consists of U^{235} and H, then a steady state will exist in which the concentration of U^{235} will be predominantly near the outer cylinder and hydrogen near the inner cylinder For a steady state, the composition of the gas being radially fed inward must be identical to that which is found at the inner cylinder and is being ejected This simplified configuration is the model we have analyzed

III Mathematical Formulation of the Problem

For the proposed magnetohydrodynamic centrifuge, it is first desirable to determine the velocity and pressure fields and power input by a single fluid analysis since, as we shall see later, the diffusion equation that governs the separation problem involves these quantities To do so, we make use of the nonrelativistic magnetohydrodynamic equations (see, for example, Ref 16) On the assumption that 1) the plasma is incompressible (compressibility is discussed later); 2) the Lorentz force is the only body force; and 3) the permeability and electrical conductivity are scalar constant quantities, the MHD equations reduce to the following (in mks units):

Continuity

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

Momentum

$$\rho(D\mathbf{V}/Dt) + \nabla p = \mathbf{J} \times \mathbf{B} + \eta \nabla^2 \mathbf{V} \tag{2}$$

Faraday's Law

$$\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t) \quad (3)$$

Ampere's Law

$$\nabla \times \mathbf{B} = \mu \mathbf{J} \quad (4)$$

Ohm's Law

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad (5)$$

Divergence Law

$$\nabla \ \mathbf{B} = 0 \quad (6)$$

where **V** denotes the fluid velocity vector, p the hydrostatic pressure, **J** the current density vector, **B** the magnetic field vector, **E** the electric field vector, ρ the bulk fluid density, σ the electrical conductivity, μ the magnetic permeability, and D/Dt the material derivative

The foregoing equations may be manipulated to yield the following:

$$\nabla \ \mathbf{V} = 0 \quad (7)$$

$$\nabla p = -\rho(D\mathbf{V}/Dt) + (1/\mu)(\nabla \times \mathbf{B}) \times \mathbf{B} + \eta \nabla^2 \mathbf{B} \quad (8)$$

$$-(1/\mu\sigma)\nabla^2 \mathbf{B} = -(\partial \mathbf{B}/\partial t) + (\mathbf{B} \ \nabla)\mathbf{V} - (\mathbf{V} \ \nabla)\mathbf{B} \quad (9)$$

$$\mathbf{J} = (1/\mu)\nabla \times \mathbf{B} \quad (10)$$

$$\mathbf{E} = (1/\mu\sigma)\nabla \times \mathbf{B} + \mathbf{B} \times \mathbf{V} \quad (11)$$

For cylindrical geometry let $\mathbf{B} = (B\ , B_\theta, B\ )$, $\mathbf{E} = (E\ , E_\theta, E\ )$, $\mathbf{J} = (J\ , J_\theta, J\ )$, and $\mathbf{V} = (u, V_\theta, w)$, where the quantities in parentheses, respectively, stand for the r, θ, and z components of the corresponding vector quantities Equations (7–11) consist of thirteen scalar equations with thirteen unknowns To obtain analytic solutions for these unknown quantities we assume that 1) the flow is axisymmetrical so that all physical quantities are independent of θ, i e , $\partial/\partial\theta = 0$; 2) the axial component of velocity w is negligibly small in comparison to the remaining components; and 3) the radial magnetic Reynolds number is small so that the azimuthal current J_θ is negligibly small, i e , $R = (\mu G_0 r_0 \sigma/\rho) \ll 1$

By the use of these assumptions it can be shown from Eqs (3, 5, and 6), that for steady-state conditions $B\ = 0$ and $B\ = B_0$, the applied magnetic field (see Ref 17) By equating components of the vector equations in Eqs (8–11) we obtain the following steady-state equations:

$$J\ = -(1/\mu)(\partial B_\theta/\partial z) \quad (12)$$

$$E\ = -(1/\mu\sigma)(\partial B/\partial z) - V_\theta B_0 \quad (13)$$

$$-\rho u\left(\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right) + \eta\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right] + \frac{\partial^2 V_\theta}{\partial z^2}\right\} + \frac{B_0}{\mu}\frac{\partial B_\theta}{\partial z} = 0 \quad (14)$$

$$\frac{1}{\mu\sigma}\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right] + \frac{\partial^2 B_\theta}{\partial z^2}\right\} = -B_0\frac{\partial V_\theta}{\partial z} + u\left(\frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r}\right) \quad (15)$$

$$\frac{\partial p}{\partial z} = -\frac{B_\theta}{\mu}\frac{\partial B_\theta}{\partial z} \quad (16)$$

$$\frac{\partial p}{\partial r} = -\rho\left(u\frac{\partial u}{\partial r} - \frac{V_\theta^2}{r}\right) - \frac{B_\theta}{\mu r}\frac{\partial}{\partial r}(rB_\theta) \quad (17)$$

From the conservation of electrical current and mass it can be shown that

$$J\ = \frac{I}{4\pi h_0 r} \qquad u = \left(\frac{G_0}{\rho}\right)\frac{r_0}{r} \quad (18)$$

in which I is the total radial electrical current input and G_0 is the radial mass flow rate per unit area for a system with porous inner and outer cylindrical walls Furthermore, for a long cylinder whose radial dimensions are much smaller than axial dimensions (i e , $r_0 - r_1 \ll 2h_0$), we neglect axial shear effects and, consequently, let $\partial V_\theta/\partial z = 0$ By substituting Eq (18) into Eqs (13–17) the following equations are obtained for a long cylinder with radial mass flow :

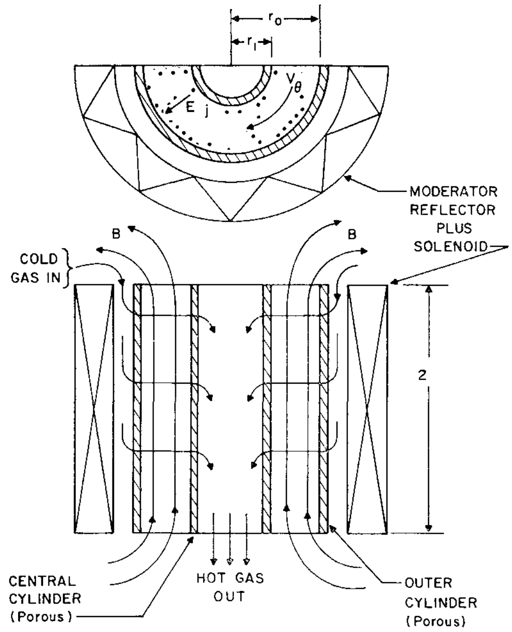

Fig 2 Simplified configuration

$$E\ = (I\ /4\pi h_0 \sigma r) - V_\theta B_0 \quad (19)$$

$$-\frac{G_0}{r}\left(\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right) + \eta\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right] - \frac{B_0 I}{4\pi h_0 r} = 0 \quad (20)$$

$$\frac{1}{\mu\sigma}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right] - \frac{G_0}{\rho r}\left(\frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r}\right) = 0 \quad (21)$$

$$\frac{\partial p}{\partial z} = -\frac{B_\theta}{\mu}\frac{\partial B_\theta}{\partial z} \quad (22)$$

$$\frac{\partial p}{\partial r} = \rho\left[\left(\frac{G_0 r_0}{\rho}\right)^2\frac{1}{r^3} + \frac{V_\theta^2}{r}\right] - \frac{B_\theta}{\mu r}\frac{\partial}{\partial r}(rB_\theta) \quad (23)$$

To complete the mathematical formulation, we require two additional equations based on a multifluid analysis The assumption of a partially ionized incompressible plasma (consisting of five components, namely, electrons, light and heavy ions, and light and heavy neutral particles) is represented mathematically by

$$\rho = \sum_{i=1}^{5} n_i m_i = \text{const} \quad (24)$$

in which n_i is the number density and m_i the mass of a particle of the ith species If we assume that the plasma is electrically neutral and that the mass of an electron is negligibly small compared to that of an ion or a neutral particle, the last equation reduces simply to

$$\rho = n_1 m_1 + n_2 m_2 = \text{const} \quad (25)$$

where n_1 is the total number density of species 1 and m_1 the mass of its particle (either ionic or neutral) In this analysis, subscript 1 refers to the atomic hydrogen and 2 to uranium 235

Finally, we write a diffusion equation for a multicomponent system whose radial mass flux is due to the coupling effect of radial concentration gradient and forces (mainly centrifugal force) causing the pressure gradient Based on the assumption that ion slip is negligibly small, it is shown in Ref 17 that the diffusion equation is

$$rD\left\{\frac{d}{dr}(n_1+n_2)+\frac{n_1m_1+n_2m_2}{kT}\left[\left(\frac{G_0r_0}{\rho}\right)^2\frac{1}{r^3}+\frac{V_\theta^2}{r}\right]\right\}=\frac{r_0G_0}{m_p} \quad (26)$$

where m_p is the average mass of a particle of the bulk plasma defined as $m_p \equiv \rho_0/n_0$ with ρ_0 and n_0 being the conditions at $r = r_0$ for the bulk plasma

The appropriate boundary conditions on V_θ, B_θ, n_1, and n_2 are[17]

$$V_\theta = 0 \text{ at } r = r_1 \text{ and } r = r_0 \quad (27)$$

$$B_\theta = 0 \text{ at } z = h_0 \quad (28)$$

$$B_\theta = \mu I/4\pi r \text{ at } z = -h_0 \quad (29)$$

$$(\partial/\partial r)(rB_\theta) = 0 \text{ at } r = r_1 \text{ and } r = r_0 \quad (30)$$

$$n_1 = n_{01} \text{ and } n_2 = n_{02} \text{ at } r = r_0 \quad (31)$$

The chemical composition of the radial mass flow injection must be the same as that found at the inner cylinder wall for the maintenance of a steady state The inner wall composition depends upon the power input, velocity distribution, etc This problem has been discussed in Ref 7

IV Solutions

Detailed solutions of Eqs (19–23) which satisfy the boundary conditions of Eqs (27–30) are, for $R_u = \mu G_0 r_0 \sigma/\rho \ll 1$ (see Ref 17 for derivations),

$$\frac{V_\theta}{\phi_0/B_0r_0} = \mathcal{R}\left[a_1\zeta^m + \frac{a_2}{\zeta} - \zeta\right] \quad (32)$$

$$\frac{p_0 - p}{\rho\phi_0^2/2B_0^2r_0^2} = \mathcal{R}^2 f_p \quad (33)$$

$$I_r = \frac{8\pi h_0 G_0 \phi_0}{B_0^2 r_0}\mathcal{R} \qquad B_\theta = -\frac{\mu I r_0}{8\pi h_0^2}\left(\frac{1}{r}\right)[z - h_0] \quad (34)$$

where

$\zeta = r/r_0$ ϕ_0 = voltage difference between electrodes

$$m = 1 + R_N \qquad a_1 = \frac{1-\zeta_1^2}{1-\zeta_1^{m+1}} \qquad a_2 = \frac{\zeta_1^2 - \zeta^{m+1}}{1-\zeta_1^{m+1}} \quad (35)$$

$$\mathcal{R} = \frac{2(m+1)}{[m+1]\left[\zeta_1^2 - 1 + 2\left(\frac{2R_N}{M^2} - a_2\right)\ln\zeta_1\right] - 2a_1(\zeta_1^{m+1} - 1)} \quad (36)$$

$$f_p = (c_4^2 + a_2^2)\left(\frac{1}{\zeta^2} - 1\right) + \frac{a_1^2}{m}(1 - \zeta^{2m}) + \frac{4a_1a_2}{m-1}(1 - \zeta^{m-1}) - \frac{4a_1}{m+1}(1 - \zeta^{m+1}) + 4a_2\ln\zeta + 1 - \zeta^2 \quad (37)$$

with

$$c_4 = \frac{\beta}{\mathcal{R}} \qquad \beta = \frac{G_0/\rho}{\phi_0/B_0r_0}$$

the ratio of the radial velocity at the outer wall to the drift velocity, $M = B_0r_0(\sigma/\eta)^{1/2}$, the Hartmann number and $R_N = G_0r_0/\eta$, the radial Reynolds number The variation of the azimuthal velocity as a function of the Hartmann number]Eq (32)] is shown in Fig 3 for a radial Reynolds number of −5. The pressure distribution [Eq (33)] is shown in Fig 4 The variation of V_θ as a function of radial mass flow G_0 is shown in Fig 5

From Eq (34) we obtain the electric power required for the rotating plasma; it is given by

$$P_e = (8\pi h_0 G_0 \phi_0^2/B_0^2 r_0)\mathcal{R} \quad (38)$$

With this expression for power, c_4^2 may be cast in the form

$$c_4^2 = \frac{\beta^2}{\mathcal{R}^2} = \left(\frac{4\pi h_0 r_0 G_0^3/\rho^2}{P_e}\right)\frac{2}{\mathcal{R}} = \frac{2}{\mathcal{R}\Pi}$$

where

$$\Pi = P_e/(4\pi h_0 r_0 G_0^3/\rho^2)$$

the ratio of the electrical power input to the energy rate of the plasma flowing at $r = r_0$ Thus, alternatively,

$$f_p = \left(\frac{2}{\Pi\mathcal{R}} + a_2^2\right)\left(\frac{1}{\zeta^2} - 1\right) + \frac{a_1^2}{m}(1 - \zeta^{2m}) + \frac{4a_1a_2}{m-1}(1 - \zeta^{m-1}) - \frac{4a_1}{m+1}(1 - \zeta^{m+1}) + 4a_2\ln\zeta + 1 - \zeta^2 \quad (39)$$

By using Eqs (23) and (25) in Eq (26) we obtain

$$\frac{d}{dr}(n_1 + n_2) + \frac{1}{kT}\frac{\partial p}{\partial r} = \frac{r_0G_0}{rDm_p} \quad (40)$$

The solutions of the last equation which satisfy the boundary conditions of Eq (31) are, for an isothermal system,

$$n_1 - n_{01} = \frac{1}{1-\alpha}\left[\frac{p_0 - p}{kT} + \frac{r_0G_0}{Dm_p}\ln\frac{r}{r_0}\right] \quad (41)$$

$$n_2 - n_{02} = \frac{\alpha}{\alpha - 1}\left[\frac{p_0 - p}{kT} + \frac{r_0G_0}{Dm_p}\ln\frac{r}{r_0}\right] \quad (42)$$

where $\alpha = m_1/m_2$ Then using the expression for the pressure given in Eq (33), we obtain the steady state species concentration Thus,

$$\frac{n_1 - n_{01}}{\rho/m_1} = \frac{\alpha}{1-\alpha}\left[\frac{m_2\phi_0^2/2B_0^2r_0^2}{kT}\mathcal{R}^2 f_p + \frac{r_0G_0m_2}{\rho Dm_p}\ln\zeta\right] \quad (43)$$

$$\frac{n_2 - n_{02}}{\rho/m_2} = \frac{\alpha}{\alpha-1}\left[\frac{m_2\phi_0^2/2B_0^2r_0^2}{kT}\mathcal{R}^2 f_p + \frac{r_0G_0m_2}{\rho Dm_p}\ln\zeta\right] \quad (44)$$

We observe that

1) $(m_2\phi_0^2/2B_0^2r_0^2)/kT = \Pi\epsilon_0/\mathcal{R}$

in which

$$\epsilon_0 = (m_2G_0^2/2\rho^2)/kT$$

the ratio of the kinetic energy of species 2, to the mean therma energy of the plasma and

2) $$\frac{r_0G_0m_2}{\rho Dm_p} = \left(\frac{\eta}{\rho D}\right)\left(\frac{G_0r_0}{\eta}\right)\left(\frac{m_2}{m_p}\right)$$

the product of the Schmidt number S_N, the Reynolds number R_N, and the mass ratio of species 2 and the bulk plasma α_p Consequently,

$$\frac{n_1 - n_{01}}{\rho/m_1} = \frac{\alpha}{1-\alpha}[\Pi\epsilon_0\mathcal{R}f_p + S_NR_N\alpha_p\ln\zeta] \quad (45)$$

$$\frac{n_2 - n_{02}}{\rho/m_2} = \frac{\alpha}{\alpha-1}[\Pi\epsilon_0\mathcal{R}f_p + S_NR_N\alpha_p\ln\zeta] \quad (46)$$

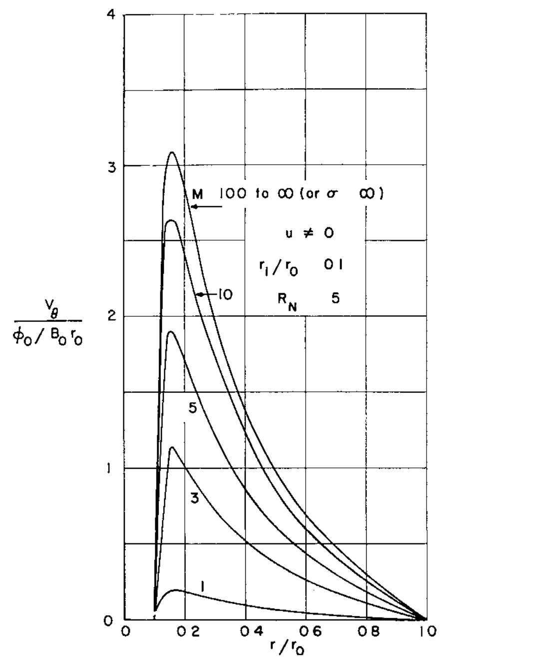

Fig 3 Variation of the rotational velocity of the viscous fluid with Hartmann number

Recognizing that

$$\rho = n_{01}m_1 + n_{02}m_2 = \rho_{01} + \rho_{02} \text{ at } r = r_0$$

we may rewrite Eqs (45) and (46) in the following alternate forms:

$$\frac{n_1}{n_{01}} = \frac{\alpha}{1-\alpha}\left[1 + \frac{\rho_{02}}{\rho_{01}}\right][\Pi\epsilon_0\Re f_p + S_N R_N \alpha_p \ln\zeta] + 1 \quad (47)$$

$$\frac{n_2}{n_{02}} = \frac{\alpha}{\alpha-1}\left[1 + \frac{\rho_{01}}{\rho_{02}}\right][\Pi\epsilon_0\Re f_p + S_N R_N \alpha_p \ln\zeta] + 1 \quad (48)$$

in which ρ_{01} and ρ_{02} are the partial densities of species 1 and 2 at $r = r_0$ Plots of the steady-state concentrations of hydrogen and uranium in a hydromagnetic centrifuge reactor are shown in Fig 6 representing Eqs (47) and (48)

By the use of the foregoing relationships, several macroscopic quantities of interest may be determined The electrical power required to rotate the plasma per unit mass is found from Eq (38) to be

$$\frac{P^m}{\eta\phi_0{}^2/\rho B_0{}^2 r_0{}^4} = \frac{4|R_N|}{1-\zeta_1{}^2}\,\Re \quad (49)$$

where $|R_N|$ stands for the absolute magnitude of the radial Reynolds number The radial Reynolds number is to be taken as negative in this analysis in which mass flows radially inward; i e , G_0(or R_N) and, consequently, I [see Eq (34)] are negative

A large amount of the electrical power input may be converted into heat by the dissipative mechanisms of joule heating and viscous mixing The rate of joule heating is given by the integral of J^2/σ over the whole volume of the device, namely,

$$Q_j = \frac{2\pi}{\sigma}\int_{z=-h_0}^{h_0}\int_{r=r_1}^{r_0}\left(\frac{I_r}{4\pi h_0 r}\right)^2 r\,dr\,dz \quad (50)$$

The rate of heating of the plasma by viscous dissipation is given by the integral

$$Q_v = \int\int\int_{\bar{v}} \eta\phi_v d\bar{V} \quad (51)$$

in which

$$\phi = 2\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2\right] + \left[r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right]^2 - \frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right]^2$$

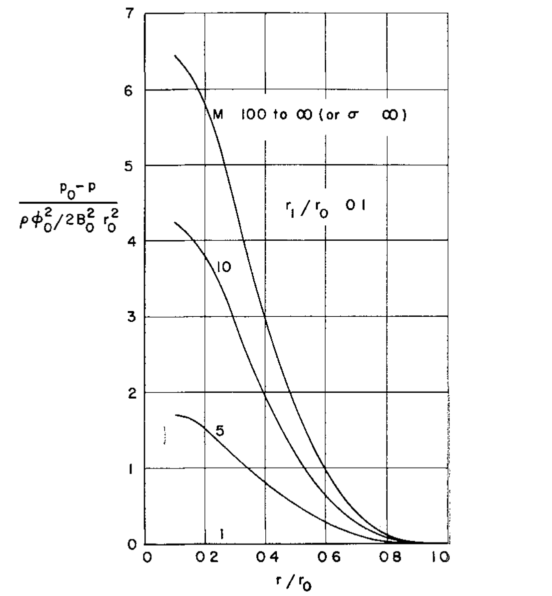

Fig 4 Variation of the pressure field of the viscous fluid with Hartmann number

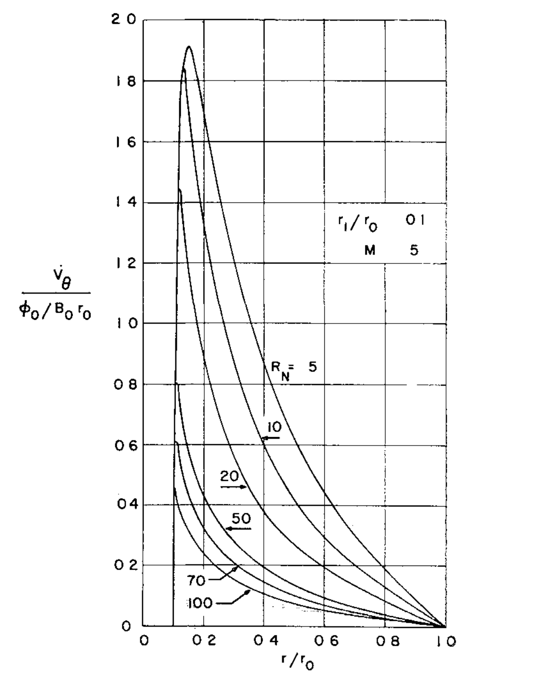

Fig 5 Variation of the rotational velocity of the viscous fluid with radial Reynolds number

The expressions for I , u, and V_θ are substituted into Eqs (50) and (51); the resulting integrals have been determined

Plots, representing Eqs (47–51), are shown in Figs 7–10, respectively, for 1) the ratio of hydrogen to uranium at the inner wall as a function of electrical power input; 2) the effect of Hartmann number upon the power required; 3) the effect of radial Reynolds number upon joule heating; and 4) the viscous dissipation rate as a function of radial Reynolds number

V Rocket Performance and Comments

A Criticality

The calculation for criticality of a nonhomogeneous gaseous reactor is very complex Besides the work of Safonov,[6] there is the work of Spencer[18] and Hyland et al [19] which bears directly on the gaseous core reactor for pro-

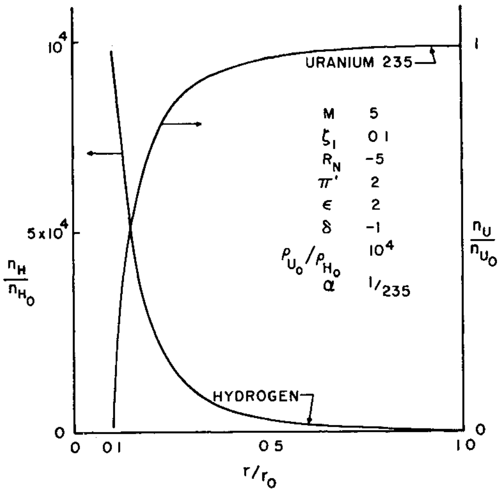

Fig 6 Concentration profiles for uranium and molecular hydrogen

pulsion Although the pressures are still high in most systems treated to date, sizes suitable for flight systems seem obtainable

The average concentration of fuel (uranium) to propellant (hydrogen) is in the range $10^{-2} \leq n_U/n_H \leq 10^{-3}$ Thus, the reactor gas consists principally of hydrogen A reflecting thermal moderator is needed to keep the pressure and size of the reactor to reasonable limits

B Reactor Gas Properties

Since the reactor gas is primarily hydrogen, many of its high-temperature properties have been determined See, for example, the work by Eisen and Gross [20] Elementary kinetic theory has been used to estimate other properties of a hydrogen plasma by Kessey and Gross [21] For example, at 1 ev temperature and 500 atm pressure, σ: electrical conductivity $= 1 \times 10^3$ mhos/m; D_{HU}: coefficient of diffusion $= 5 \times 10^{-4}$ m²/sec; and η: coefficient of viscosity $= 1 \times 10^{-3}$ kg/m-sec

C Rocket Performance

If we assume that the rocket exhaust is, as it must be, primarily hydrogen, the specific impulse is primarily a function of stagnation temperature The specific impulse for both frozen and equilibrium flow of hydrogen has been computed by Rom et al [8] At 11,000°K (nearly 1 ev) a specific impulse of about 3000 sec should be obtainable Configurations of practical interest have been examined by Kessey and Gross[21] and typical results for a hydromagnetically driven vortex gaseous nuclear rocket system are as follows: exhaust gas temperature, 11,600°K; average reactor gas pressure, 500 atm; reactor volume, 10 7 m³; mass of uranium, 15 kg; reactor total power, 1 21 $\times$ 10⁶ kw; reactor radius, 1 0 m; radial electrical current, 348 amp; voltage difference across vortex, 50 v; axial magnetic field, 10⁴ gauss; exhaust composition (n_H/n_U), 1 8 $\times$ 10⁴; mass flow, 5 63 lb/sec; thrust, 16,900 lb; average hydrogen-uranium particle ratio (n_H/n_U), 108; Hartmann number, 1000; and radial Reynolds number, -115

The small mass flow is the result of theoretical limitations upon the permissible radial Reynolds number [21] Although weight estimates for such a system are not available, it appears that thrust to weight ratios of the order of 0 1 or larger might be obtained Clearly, very large gains are obtainable if we can go to the high temperatures that can be generated from a gaseous nuclear reactor

D Compressibility and Turbulence

The effects of compressibility are certainly of importance, but in these calculations they are, for simplicity, ignored However, for the pure gas centrifuge the effect of compressibility has been considered by Kerrebrock et al [7]

The effect of turbulence has hindered greatly the effect of separation in a gas centrifuge [11–13] The hydromagnetic centrifuge discussed here will have a "stiffening effect" by the axial magnetic field that may delay the onset of turbulence However, the transition Reynolds number is unknown and can only be determined by experiment

E Secondary Flows

It is well known that in such rotating flows, secondary flows can play at times a dominant role How long is a "long cylinder," really requires further analysis and experimentation If significant axial velocity and cells form from

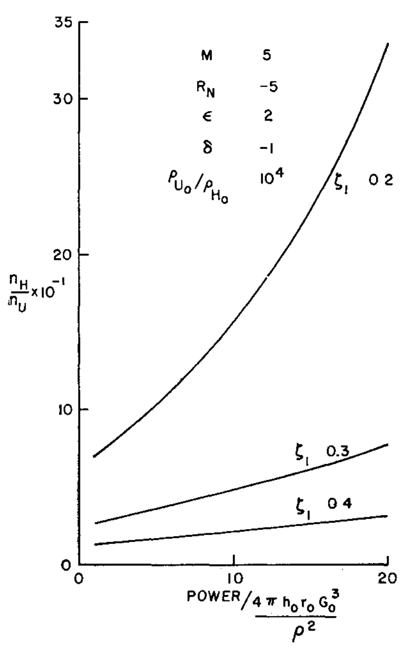

Fig 7 Concentration ratios at inner wall

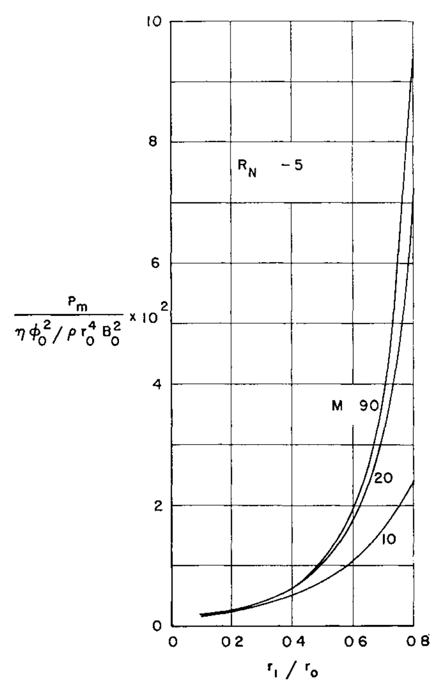

Fig 8 Variation of electrical power per unit mass with Hartmann number

secondary flow in this hydromagnetic case, it will be a serious problem Further work on the stability of the hydromagnetically spun vortex is needed Here again, the axial magnetic field will provide a stabilizing effect

F Electrical Power Requirement

The effect of electrical power input on the gas separation capability of the system was shown in Fig 7 There is also a practical electrical weight-power limitation on a flight rocket system For example, at 1 ev temperature in the reactor, one can obtain about 3000 sec impulse This is an exhaust velocity of about 5 ev per particle of hydrogen The dissociation energy of hydrogen is about 4 5 ev per molecule The exhaust swirl energy ($\frac{1}{2}mV_\theta^2$) is determined by the electrical power input and the hydromagnetics of the problem However, if we assume an electrical power plant weight of of 10 lb/kw, then the swirl velocity is a function of the ratio of the electrical plant weight to the total rocket thrust For the 3000 sec impulse case, if the electrical power plant weight is 1% of the thrust, a swirl velocity up to about 10^4cm/sec is obtainable At 3000 sec impulse, exhaust energy is 5 ev/particle (3×10^6 cm/sec); dissociation energy, 4 5 ev/molecule; and swirl energy, $0\ 5 \times 10^{-4}$ ev/particle (10^4 cm/sec, 10 lb/kw, electrical weight = 1% thrust) Although by far the majority of reactor power goes into dissociating hydrogen and the kinetic energy of the rocket exhaust, the weight of the electrical plant required to swirl the gas is still significant

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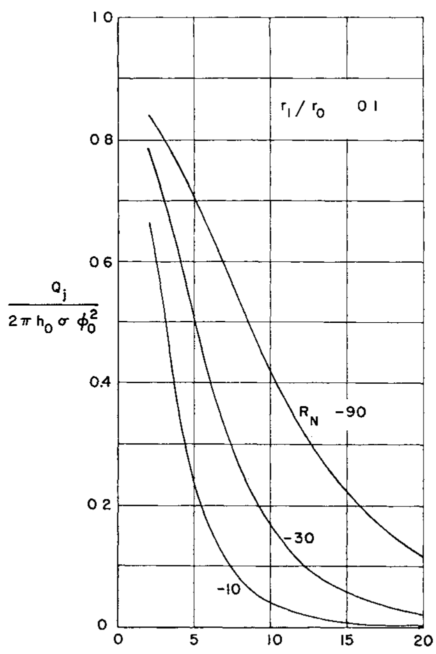

Fig 9 Variation of joule heating rate with radial Reynolds number

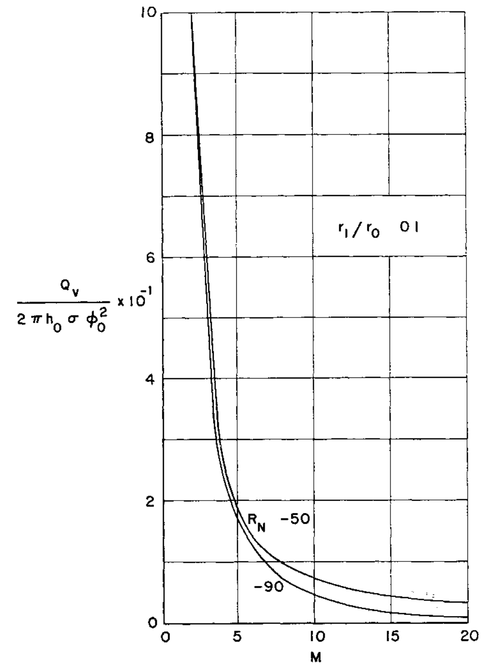

Fig 10 Variation of viscous dissipation rate with radial Reynolds number

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